SOLUTIONS

Part 1.

1.1) From the plot on the right we can get the equation of the straight line as

$$y = -0.76 \cdot x + 7.32$$ [3 points]

which taking to the power of 10 is equivalent to: $f_g = 10^{7.32} M_*^{-0.76}$, being f_g the gas fraction. Multiplying by the stellar mass we finally get:

$$M_{gas} = 10^{7.32} \times M_*^{0.24}$$

a = 10^7.32 = 2.09 E7 [1 point] ; b = 0.24 [1 point]

No tolerance allowed for these values

1.2) [15 points]

V	LogV	K	Zx	Zy	ZxZy
79.4	1.8998	-16.8	-1.1778	1.3873	-1.6339
100.1	2.0004	-19.2	-0.7831	0.4970	-0.3892
158.5	2.2000	-21.3	-0.0001	-0.2819	0.0000
251.2	2.4000	-21.4	0.7845	-0.3190	-0.2502
316.2	2.5000	-24	1.1765	-1.2834	-1.5100
Ave	2.2001	-20.54		sum ZxZy	-3.7833
St. Dev	0.2549	2.696		r	-0.9458

The best-fit straight line is $K = -10 \cdot log_{10}(V_{max}) + 1.47$.

A correct answer does not need to show step-by-step calculations.
Tolerance: +/- 0.01.
Partial score if the full answer is not correct: Slope: [3 points] ; Intercept: [2 points] ; calculations: if every single number is right except the final answer [10 points] ; if at least half the numbers are correct [5 points]. Tolerance in each number: +/- 0.01

Part 2.

2.1) [10 points]

Galaxies are at the same distance, therefore the difference in apparent magnitudes is the same as in absolute magnitudes, and this relates to a luminosity ratio:

$$(k_1 - k_2) = (K_1 - K_2) = -2.5 \times log_{10}(\frac{L_1}{L_2})$$ [2 points]

For having the same stellar populations, and ignoring interstellar extinction, the luminosity of each galaxy is proportional to its stellar mass with the same mass-to-light ratio for both galaxies, such that:

$$(k_1 - k_2) = -2.5 \times log_{10}(\frac{M_{*1}}{M_{*2}})$$ [2 points]

The difference in magnitudes being 6, we obtain:

$$(\frac{M_{*1}}{M_{*2}}) = 10^{2.4}$$ [2 points]

No tolerance allowed in the exponent value.

2.2) [4 points]

Using the last result in combination to that of 1A:

$$\frac{M_{gas1}}{M_{gas2}} = \left(\frac{M_{*1}}{M_{*2}}\right)^{0.24}$$ [2 points]

$$\frac{M_{gas1}}{M_{gas2}} = 10^{0.576}$$ [2 points]

Exponent approximated to 2 digits is OK.

2.3) [6 points]

We found the slope of the TF relation in 1B: $\frac{\Delta K}{\Delta \log(V_{max})} = 10$, with $\Delta K = 6$ for our galaxies: $\frac{V_{max1}}{V_{max2}} = 10^{0.6}$ [2 points]. Now consider the mass distribution as approximately spherically-symmetric, giving:

$$V_{max} = \sqrt{\frac{G \cdot M_{tot}(<R_{max})}{R_{max}}},$$ [2 points]

which combined with the ratio just found two lines above leads to:

$$\frac{M_{tot1}}{M_{tot2}} = 10^{1.2}$$ [2 points]

No tolerance allowed in the exponent value.

Part 3. [15 points]

Baryonic mass is $M_{Baryonic} = \frac{4.39\times10^{11}}{7.82}\ [M_\odot]$ and Dark matter mass is given by $M_{dm} = 6.82 \times \frac{4.39\times10^{11}}{7.82}\ [M_\odot]$. The baryonic contribution needs to be decomposed into stellar and gaseous masses, which is not doable analytically but must be faced numerically, by tuning the numbers, knowing their sum and knowing the relation between them from 1.1. The final table looks like this:

Galaxy	Apparent magnitude k	$M_{gas}\ [M_\odot]$	$M_*\ [M_\odot]$	$M_{dm}\ [M_\odot]$	$M_{tot}\ [M_\odot]$
G_1	19.2	7.67×10^{9}	4.85×10^{10}	3.83×10^{11}	4.39×10^{11}

Equation to be solved: $1 - k = 0.1415\, k^{0.24}$ This gives k = 0.8634

5 points for each value, with a tolerance of 5%

If does not score for Mg, M*, but has a correct equation , 5 points

Part 4

4.1) [4 points]

We basically found in 2.1 that $\left(\frac{M_{*1}}{M_{*2}}\right) = 10^{\frac{\Delta K}{-2.5}}$, being $\Delta K = -6$. In this new case, we have the extreme values of ΔK to be -6.4 and -5.6, therefore the extreme values for the exponent are these values divided by -2.5, then $e \in [\,2.24\,,\ 2.56\,]$

No tolerance allowed in limits of the interval.

4.2) [10 points]

We have to invert the equation from 1.2 to get: $log_{10}(V_{max}) = -\frac{K}{10} + 0.1467$. Now we can get the difference between

the measured values and linear-fit predictions, approximating numbers to the second digit:

$K[mag]$	$V_{max}\,[km/s]$	$log_{10}(V_{max})$ measured	$log_{10}(V_{max})$ from linear fit	$\lvert\Delta\, log_{10}(V_{max})\rvert$
-16.8	79.4	1.9	1.83	0.07
-19.2	100.1	2.0	2.07	0.07
-21.3	158.5	2.2	2.28	0.08
-21.4	251.2	2.4	2.29	0.11
-24.0	316.2	2.5	2.55	0.05

Taking two times the RMS of the residuals: $\sigma_{stat} = 0.157$

A correct answer does not need to show step-by-step calculations.
If the final value is not correct, up to [5 points] for partial calculations: [1 point] for each value of $log_{10}(V_{max})$ correctly calculated.
Tolerance in each number: +/- 0.01

4.3) With the uncertainties, the expression $log_{10}(V_{max}) = -\frac{K}{10} + 0.1467$ becomes:

$log_{10}(V_{max}) = -\frac{K}{10} + 0.1467 \pm \frac{0.2}{10} \pm 0.157$ [2 points]

Values approximated to the second digit are accepted.

Writing the same expression down for galaxy 1 and 2, subtracting, and using $\Delta K = -6$ we get:

$log_{10}(\frac{V_{max1}}{V_{max2}}) = 0.6 \pm 0.02 \pm 0.157 \pm 0.02 \pm 0.157$ [4 points]

Which leads to the extreme values: 0.954 and 0.246. [2 points]

Finally remembering that circular velocities scale as the square root of the total mass, we have to multiply this exponent by 2, getting the final answer:

$g \in [0.49,\ 1.91]$ [2 points]

A tolerance of +/- 0.02 is acceptable.

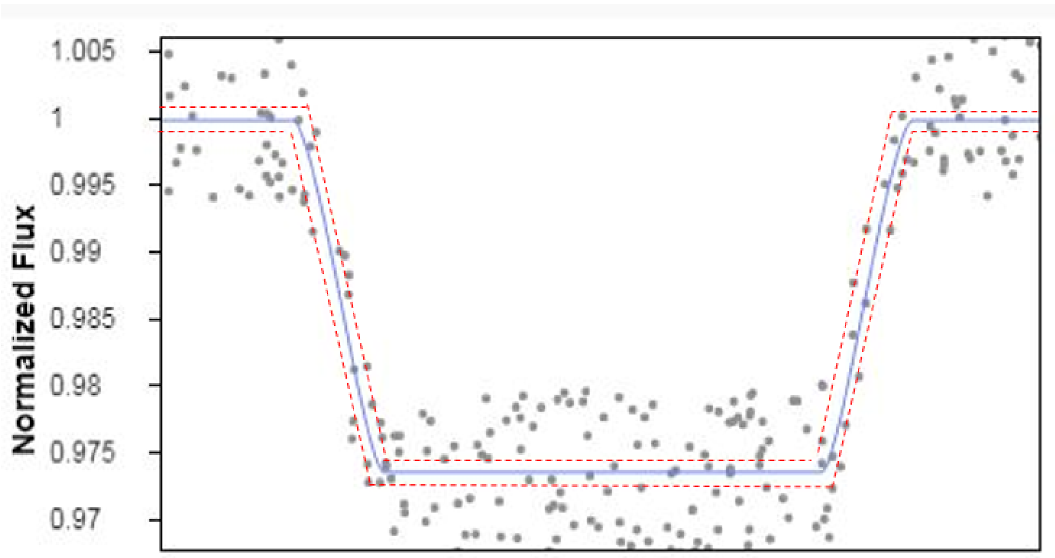
SOLUTIONS

Part 1. [20 points]

- With the parallax we get the distance: $D = \frac{1}{parallax} = 48.38\ pc$, [1 point]
  then we get the absolute magnitude using the distance modulus $M_V = m_v - 5\ log_{10}(D) + 5 = 4.2067$, [1 point] and finally we go for the luminosity as BC is assumed to be the same for all F/G stars, $(M_V - M_{V_\odot}) = (M_{bol} - M_{bol_\odot})$ [2 points]
  by comparing with the magnitude of the Sun: $\frac{L}{L_\odot} = 10^{-0.4(M_V - M_{V_\odot})} = 1.759$. [1 point]
- We find the stellar radius from the Stephan-Boltzmann law, which in solar units leads to $\frac{T_{eff_\star}{}^4}{T_{eff_\odot}{}^4} = \frac{R_\odot{}^2}{R_\star{}^2}\frac{L_\star}{L_\odot}$ [2 point], i.e., $R_\star[R_\odot] = \frac{T_{eff_\odot}{}^2 \sqrt{L_\star[L_\odot]}}{T_{eff_\star}{}^2} = 1.238$. [2 points]
- Now to find the stellar mass consider the surface gravitational acceleration, which for Gauss' law and spherical symmetry considerations is: $g = G\frac{M_\star}{R_\star{}^2}$, [1 point] then one solves $M_\star$ for and puts it in the right units, $M_\star = 1.944\ [M_\odot]$ [3 points]
- Planet's orbital radius comes from $a = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = 0.0565\ A.U$ [3 points]
- Finally, checking the dimming due to the transit this is of approximately 2.6%, which is give by the ratio between the areas of the planet disk and the stellar disk, i.e., $\left(\frac{R_p}{R_\star}\right)^2 = 0.026$, [2 points] leading to $R_p = 1.296\ [R_J]$ [2 point]

Luminosity of the star	Radius of the star	Mass of the star	Mean planet's orbital radius	Radius of the planet in Jupiter's radius
$L_\star[L_\odot]$	$R_\star[R_\odot]$	$M_\star[M_\odot]$	$a[au]$	$R_p[R_J]$
1.759	1.24	1.94	0.0565	1.296

Tolerance: of 5% on the final answers is acceptable, numbers approximated to the second digit in the partial calculations are OK.



Part 2. [25 points]

2.1) [15 points]

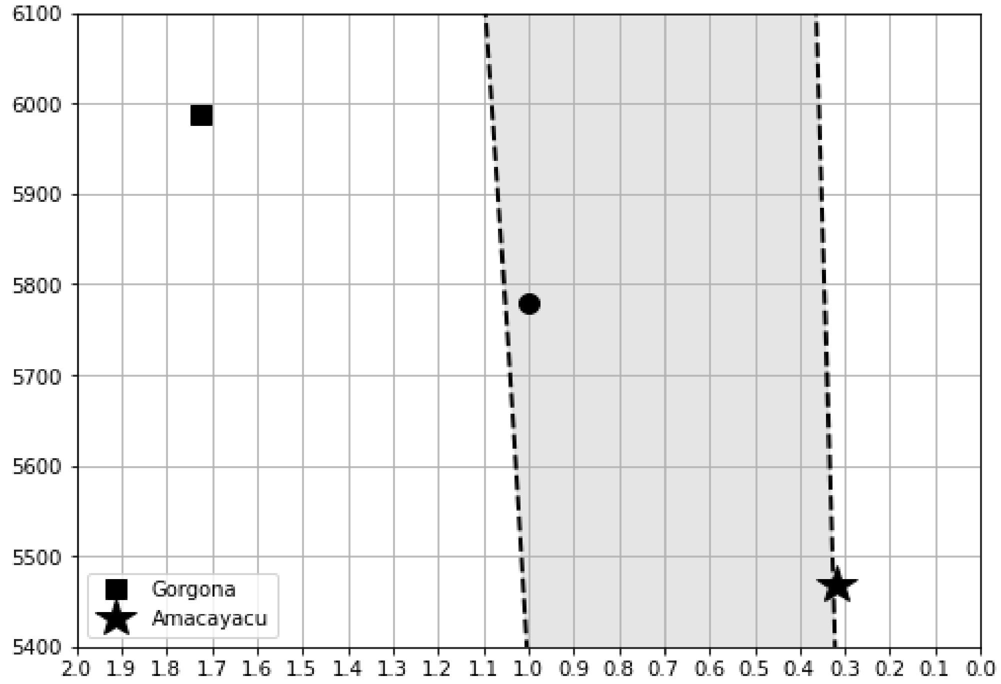
One first needs to calibrate the axes. The shown circle representing the Earth gives as a vertical mark in 5778 °K, and a horizontal one at $S_{eff} = 1$. Then one need to retrieve additional data from one or more of the additional observations:

- Having the temperature mark for the Sun, one can estimate S_{min}, adding a second mark on the horizontal axis, so the scale of this one can be calibrated. Then one can proceed backwards, starting from a given value of S_{min} or S_{max}, find the corresponding temperature, so the vertical axis can be calibrated as well.
- Alternatively, one can notice that the limiting S_{max} curve crosses the X axis in $S_{eff} = 1$, and look numerically for the corresponding temperature of the bottom line, thus calibrating the vertical axis. Then starting from a given temperature, one calibrates de horizontal one pivoting on S_{min} / S_{max} calculations as explained before.

  There are several ways, some more precise than others. The student should be as careful as possible (we give small tolerance to the error in the calibrations of the axes). A tricky part is that the X-axis increases towards the left, but this is something rigorous students will discover, as there is no self-consistency otherwise. Intuition may also help if one knows that Earth is closer to the high-flux end of the Sun's habitable zone.

- The luminosity of Gorgona's host star was found in Part A. After finding the luminosity of Amacayacu's one the final plot looks like this:

For placing Gorgona in the right position [2 pt]



For calculating flux and placing Amacayacu in the right position [4 pt]

Grading Scheme:
TOTAL: 15 pts
Points should be given to partial achievements like:

- Horizontal axis - [4 points] – sun position - [1 point] (if did not get the 4 points)
- Vertical axis - [5 points] – sun position - [1 point] (if did not get the 5 points)

Tolerance for the position of the planets: +/- 0.025 in Seff ; +/- 25 [K] in Teff

2.2) [10 points]

NONE of the exoplanets lie in habitable zones. They orbit their stars too close, so the effective flux is many times larger than S_{max} . The long-but-straightforward answer is to perform all the calculations. The elegant, clever one, is to note that Amacayacu orbits the faintest/coolest star, and it is the one farthest away, so its effective flux is the absolute minimum of the whole set. This effective flux was calculated as ~0.3 in A), assuming a distance of 1 A.U. With the real distance, 0.08 A.U, it grows by a factor $\frac{1}{0.08^2}\sim 156$, i.e., it has $S_{eff}\sim 46$, but the habitable zone for this range of temperatures can not exceed $S_{eff}\sim 1.1$, which completely closes the case.

Name	Seff
Tayrona	1785.994090
Iguaque	1499.270574
Gorgona	1096.175314
Amacayacu	50.794890
Malpelo	1374.231792
Pisba	5568.747040
Tatama	678.730709

Grading Scheme:

- [9 points] If the student argues about the case of Amacayacu as the limiting case, concluding that none of the exoplanets lie in the HZ, even if the flux is wrongly calculated. [1 point] additional for getting Amacayacu's Seff with 10% error or less.

If the student applies an exhaustive solution:

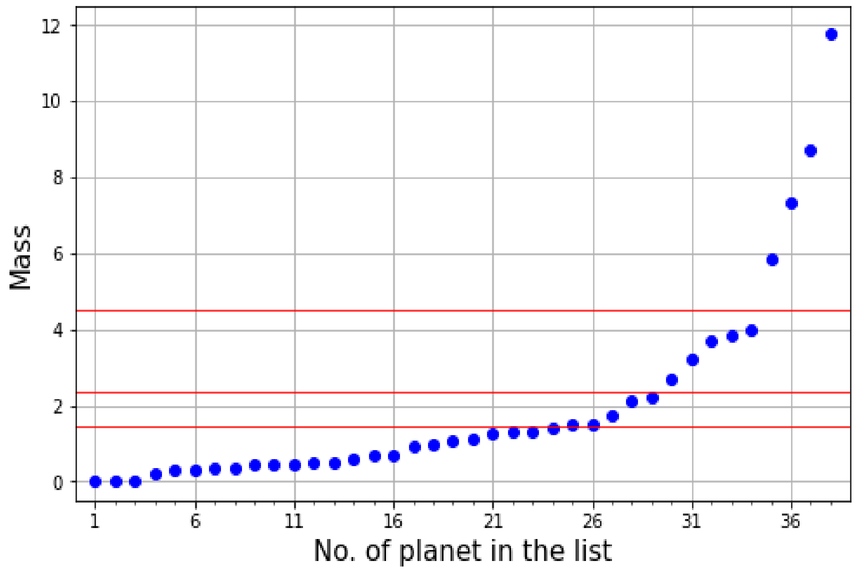
- Each effective flux must be calculated correctly up to a 10% , giving [1 point] per each Seff. [3 pts] for explaining that all these fluxes are way too much for the HZ, which can only be awarded if all the 7 fluxes are correct. This is because it is pointless to argue about calculating numbers and comparing, if you do not calculate well.

Part 3.

3.1) [3 points] per each row. [1 point] extra for the only number in the last row.

Low-mass sample	Sample size	μ	σ	μ + σ	No. of planets to exclude
Full / Original	38	1.99	2.55	4.54	4
1st subsample	34	1.23	1.11	2.34	5
2nd subsample	29	0.84	0.61	1.45	5
Final subsample	24	-	-	-	-

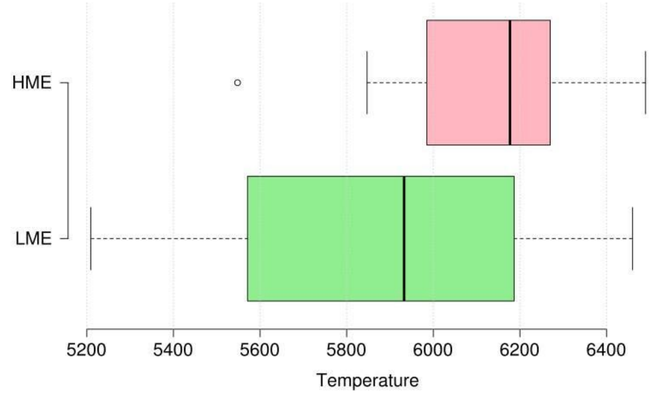
3.2) Graph plotting technique: Coverage [1 point], labelling [1 point], correct plotting [1.5 point], horizontal lines [0.5 points x 3]



3.3) [1.5 points] for each quartile, [1 point] for each median. Tolerance 5%. 0.5 for each min and max

T_{eff}	Min.	1st Quartile	Median	3rd Quartile	Max
LME	5209	5574.5	5932.5	6181.75	6460
HME	5548	5992.5	6177.0	6255.0	6490

3.4) Boxplot [1.5 points] x 2, outlier [1 point]



Yes, the boxplots indicate that HMEs tend to get formed around hotter stars. [1 point]